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(Build. 1, 2, Glushkov Prosp., Kyiv 03680, Ukraine; e-mail: machur@ukr.net)**PARTICLE THERMALIZATION ENTROPY AND UNRUH EFFECT**

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*We propose the method for estimation of entropy generated during the string breaking in high energy collisions. The approach is highly based on the ideas proposed by Kharzeev D et al and may be useful in thermalization problem.*

*Keywords:* Ентропія зчепленості, термалізація, ефект Унру/Entanglement entropy, thermalization, Unruh effect

**1. Introduction**

Quark-gluon plasma, once being born in heavy ion collisions, quickly (by the time of order of 1 fm) forgets about its initial conditions and results in a completely thermal state. Loss of any information but except of some total quantities (such as energy, angular momentum, charge etc.) implies that the system meets with generation of large amount of entropy right after the collision. To date there is no complete theoretical explanation of this process known as ‘thermalization’.

In [1] authors proposed the description of thermalization which is based on the Unruh [2] effect. They proposed to consider Unruh temperature  $T_U$

$$T_U = \frac{a}{2\pi} \quad (1)$$

as the Hagedorn temperature  $T_H$  (all over the paper we use planck units); here  $a$  is the acceleration caused with the momentum exchanges during the collision.

The idea has been further developed in [3, 4]. Authors suggested that during the collisions string breaking leads to parton deceleration  $a$  and, consequently, to appearance of the horizon. The deceleration can be expressed in a simple form via the string tension  $\sigma$ . Due to the Unruh effect, see (1), one meets with particle emission at the horizon. The partons have no access to the region under the horizon thus meeting with the information loss and, consequently, thermalization. The color horizon (CH) radius, which was named as color confinement radius, turned out to be of the order of magnitude. Comparison of the model predictions to experimental data one can find in [5].

However, the presence of horizon implies the existence of some kind of black hole. Theoretical analysis of this idea and search for the corresponding black hole type was made in [6].

Further development of the Unruh thermalization mechanism one can find in [7] where author considers P and CP violation via Unruh mechanism. In [8] the

discrepancy between the approach and the RHIC data on photon radiation was revealed.

One may conclude that the Unruh thermalization is a promising approach. However it can not explain completely all the data and therefore needs further research and verification.

Here we utilize the analogy, proposed in the papers mentioned above, between the CH generated in high energy collisions and the event horizon (EH) of Schwarzschild black hole (BH). Our idea is highly related to [9, 10] where scalar radiation entropy from EH was considered from the Schmidt decomposition viewpoint. Here we demonstrate how BH entropy can be applied for the analysis of spinless radiation from CH – as it is “seen” by the particles themselves. The approach might shed more light on the thermalization problem.

The paper is organized as follows. In section 2 we present the basics of the model. Section 3 is devoted to the asymptotics of the scalar radiation entropy at CH. Conclusions and open questions one can find in section 4.

**2. Formalism**

The approach is based on the model presented in [9, 10]. In this section we utilize the key formulae and concepts from the papers.

Let consider what happens during high energy collision; consideration is restricted to spherically symmetric case for simplicity.

Right after the collision the partons possess large momenta, thus leading to the string breaking and to the generation of new particles. The generation requires energy and therefore leads to the loss of momenta with the initial partons and to their deceleration. In [3, 4] authors proposed to interpret such a string breaking in the framework of Unruh effect and resulted in the CH appearance; the idea has been developed further in [5, 7, 8].

Due to the Unruh effect, any acceleration is equivalent to the presence of some EH and therefore implies the entropy generation.

To proceed one should analyze the interconnections between the acceleration and event horizon in more details.

Let consider Schwarzschild BH with mass  $M$ . As it follows from [2], vacuum is not invariant with respect to different frames of reference. The annihilation and creation boson operators for free-falling observer  $b$  and  $b^\dagger$  can be expressed with the help of Bogoljubov transformations as

$$\begin{aligned} b &= \frac{1}{\sqrt{1-\zeta^2}} c_{\text{out}} - \frac{\zeta}{\sqrt{1-\zeta^2}} c_{\text{in}}^\dagger \\ b^\dagger &= \frac{1}{\sqrt{1-\zeta^2}} c_{\text{out}}^\dagger - \frac{\zeta}{\sqrt{1-\zeta^2}} c_{\text{in}} \end{aligned} \quad (2)$$

where  $c, c^\dagger$  are the annihilation and creation boson operators in the accelerated frame of reference, subscripts  $\text{in(out)}$  determine the region in(out)side the BH horizon, and  $\zeta$  is defined as

$$\zeta = \exp(-4\pi M\omega), \quad (3)$$

where  $\omega$  is the energy of the quanta generated at the horizon.

Using (2) one can rewrite the vacuum state  $|0\rangle: b|0\rangle = 0$  from the free-falling frame of reference as

$$|0\rangle = \sqrt{\frac{1-\zeta^2}{1-\zeta^{2N}}} \sum_{n=0}^{N-1} \zeta^n |n\rangle_{\text{in}} |n\rangle_{\text{out}},$$

where  $N$  stands for the dimension of the in(out)side Hilbert subspace. As one can notice, this is just the Schmidt decomposition of the vacuum state. The subscript  $\text{in}$  denotes the inside degrees of freedom which are located under the EH and thus are inaccessible for the accelerated observer. Tracing out over them one results in a mixture state described with density matrix  $\rho_{\text{out}}$

$$\rho_{\text{out}} = \text{Tr}_{\text{in}} |0\rangle\langle 0| = \frac{1-\zeta^2}{1-\zeta^{2N}} \sum_{n=0}^{N-1} \zeta^{2n} |n\rangle_{\text{out}}\langle n|$$

with von Neumann entropy  $\sigma(N, \zeta)$

$$\sigma(N, \zeta) = -\ln \frac{1-\zeta^2}{1-\zeta^{2N}} - \left( \frac{\zeta^2}{1-\zeta^2} - N \frac{\zeta^{2N}}{1-\zeta^{2N}} \right) \ln \zeta^2. \quad (4)$$

Expression (4) defines the entropy for some mode with angular momentum and energy  $\omega$  being fixed, and therefore we should sum up the contributions of all the modes available.

The angular momentum of the radiated quanta is restricted with

$$0 \leq \sqrt{l(l+1)} \leq \sqrt{L(L+1)} = rp = 2M\sqrt{\omega^2 - m^2},$$

where  $m$  is the rest mass of the particles radiated away from the horizon. Summing over the angular degrees of freedom gives then

$$\begin{aligned} \sum_{l=0}^{L=1} \sum_{\mu=-l}^{\mu=l} 1 &= 4M^2 (\omega^2 - m^2) + \\ &+ \frac{\sqrt{16M^2 (\omega^2 - m^2) + 1} + 1}{2}. \end{aligned}$$

Integrating over all the  $\omega$  possible, finally we obtain the scalar entropy  $S(N, M, m)$

$$\begin{aligned} S(N, M, m) &= \frac{M^2}{6\pi^3} \int_{\zeta_m}^{\zeta_M} d\zeta \frac{\sigma(N, \zeta)}{\zeta} \left( \frac{\ln^2 \zeta}{2\pi^2} + 1 - \right. \\ &\left. - 8M^2 m^2 + \sqrt{\frac{\ln^2 \zeta}{\pi^2} + 1 - 16M^2 m^2} \right), \end{aligned} \quad (5)$$

where  $\zeta_m = \exp(-4\pi Mm)$ ,  $\zeta_M = \exp(-4\pi M^2)$ , and  $\zeta$  is defined in (3).

Entropy  $S(N, M, m)$  strongly depends on its arguments. The integrand in (5) is highly correlated to the boundaries of integration, and it is not easy to calculate  $S(N, M, m)$  even numerically. In order to estimate the entropy and to compare it to the experimental data from the collision experiments one should know its dependence on the asymptotic values of the arguments.

### 3. Asymptotic analysis

In this section we present the asymptotics of (5). Those who are interested in details we refer to [9, 10].

Before we proceed let us make some general restrictions.

As it follows from [3, 4], Unruh temperature  $T_U$  can be expressed as  $T_U = \sqrt{\sigma/2\pi}$ , where  $\sigma$  is the string tension. Using (1) one obtains then

$$M = \frac{1}{4\sqrt{2\pi\sigma}}, \quad (6)$$

where we have used that for BH  $a = 1/(4M)$ .

For the mass  $m$  of the particles generated at the CH we have

$$10^{-23} \leq m \leq 10^{-17}, \quad (7)$$

where  $10^{-23}$  stands for the electron mass, and  $10^{-17}$  stands for the mass of the Higgs boson.

One should keep in mind that masses  $M$  and  $m$  from (6) and (7) are not independent: since BH can not emit the particle with mass larger than its own then from (6)

$$m \leq M \Rightarrow 4\sqrt{2\pi\sigma} m \leq 1. \quad (8)$$

Entropy (5) is valid for the scalar radiation, i.e. for the spinless particles.

### 3.1. Finite $N$

In case of small BH, i.e.  $M \leq 1$ , the amount of degrees of freedom involved in the process will be small too. As a result  $N$ , i.e. the dimension of the corresponding Hilbert space, will be not very large.

Asymptotic expression for entropy  $S(N, M, m)$  in case of finite  $N$  was obtained in [10]. The corresponding asymptotics was assumed to be valid for the (sub)planck masses, i.e. for  $M \leq 1$  (rigorous analysis reveals that this limitation on  $M$  is inessential). The corresponding entropy  $S(N, M, m)$  can be expressed then as

$$S(N, M, m) \approx -\frac{M^2}{6\pi^2} \left( \sum_{n=1}^N \alpha_n - N\alpha_N \right) \Big|_{\omega=m}^{\omega=M}, \quad (9)$$

where

$$\begin{aligned} \alpha_n = & \frac{\zeta^{2n}}{n} \left[ 1 + 4M\omega + 8M^2(2\omega^2 - m^2) + \right. \\ & + 32M^3\omega(\omega^2 - m^2) + \\ & + \frac{1 + 6M\omega + 8M^2(2\omega^2 - m^2)}{\pi n} + \\ & + \left. \frac{3 + 16M\omega}{4\pi^2 n^2} + \frac{1}{2\pi^3 n^3} \right] + (1 - 8M^2 m^2) \times \\ & \times (2\pi - 1/n) e^{2\pi n} \text{Ei}[-2\pi n(1 + 4M\omega)], \end{aligned}$$

where  $\text{Ei}(x) = \int_{-\infty}^x e^{-t}/t dt$ ;  $M$  is defined from (6).

To obtain (9) the integrand in (5) was decomposed into series. These series are convergent for any values of  $N$ ,  $M$  and  $m$  till (8) is valid and till  $m \neq 0$ . But such weak limitations may lead to large number of terms in the sum from (9). It follows from the decomposition of expression (4) into power series up to  $\zeta^{2N}$ . To provide quick convergence of the series and to reduce the number of terms to be taken into account we can use

$$mN\sqrt{2\pi/\sigma} > 1,$$

where we used (6). This inequality can be considered as a kind of (weak) restriction for  $N$ .

In addition, as it follows from [10], the following inequality should be valid:

$$\frac{m^2(1 + 2m)^2}{32\pi\sigma(1 + m/\sqrt{2\pi\sigma})^4} \ll 1, \quad (10)$$

where we used (6). This is necessary to simplify the square root term in the integrand in (5) (see [10], eq.(8) and the text right below it). As it follows from (8), (10) is valid by default; however, it is necessary to provide good accuracy for the entropy estimation.

### 3.2. Infinite $N$

In the collision experiments one usually results in large amount of particles generated under the collision. It implies that the dimension of the corresponding Hilbert subspace  $N$  is large enough to neglect the

higher powers of  $\zeta$  ( $\zeta < 1$  in case  $m \neq 0$ , see (3)) and thus usually we can put  $N$  to be infinite. In such a case from (4)

$$\sigma(N \rightarrow \infty, \zeta) \approx -\ln(1 - \zeta^2) - \frac{\zeta^2}{1 - \zeta^2} \ln \zeta^2. \quad (11)$$

Such an asymptotics requires the collision energy  $\sqrt{s}$  to be large, since otherwise the initial momenta of the partons will be small to generate enough particles to provide the large dimension of the Hilbert space.

In such a case we consider two possible cases:  $Mm \ll 1$  and  $Mm \gg 1$ .

For  $Mm \ll 1$  we use eq.(18) from [9]:

$$\begin{aligned} S(N \rightarrow \infty, \sigma, m) & \approx 1.825 \times 10^{-2} M^2 \times \\ & \times (1 - 4.348 M^2 m^2) = \\ & = \frac{1.8 \times 10^{-4}}{\sigma} \left( 1 - 0.043 \frac{m^2}{\sigma} \right), \end{aligned} \quad (12)$$

where we used (6) also.

For  $Mm \gg 1$  we can take eq.(22) from [9]:

$$\begin{aligned} & -\frac{4(2\pi - 1)}{81\pi^3} M^4 m^2 e^{-8\pi Mm} \leq \\ & \leq S(N \rightarrow \infty, M, m) \leq \frac{4}{81\pi^3} M^4 m^2 e^{-8\pi Mm}. \end{aligned}$$

Due to the exponential boundaries the entropy in such an asymptotics can be set to 0 with high accuracy, and therefore this case can be neglected.

## 4. Discussion and conclusions

In the paper we determined the entanglement entropy generated at the CH during high energy collisions. It is estimated in the frame of reference of the interacting particles meeting deceleration due to the string breaking mechanism. To date it is applicable for the spinless radiation only.

The model utilizes spherical symmetry, so the analysis is restricted with central ion or with particle-antiparticle collisions. The last ones are preferable since the underlying formalism is based on the Schwarzschild black hole, which have zeroth electrical charge. To satisfy the electrical neutrality of CH one may consider zeroth-charge particles as the outgoing radiation since otherwise we will go outside the electrical neutrality. But such a restriction seems to be irrational due to the experimental restrictions: usually it is hard to collect data for neutral particles (take  $\pi^0$  meson for example).

In order to extend the amount of data available for the analysis one can take into account the charged outgoing particles. In such a case the CH neutrality is satisfied if one deals with large momenta exchanges, since this implies a large amount of particles being generated; then the total charge will be 0 on average.

The presented approach is an attempt of further development of the ideas proposed by Satz H et al. The presented expressions allow direct estimation of the

entropy and therefore might be useful in the thermalization problem. Surely, the need experimental verification which is the topic of our further research.

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## ЕНТРОПІЯ ТЕРМАЛІЗАЦІЇ ЧАСТИНОК І ЕФЕКТ УНРУ

### Резюме

Запропоновано метод оцінки ентропії, яка генерується під час розриву струни у високоенергетичних зіткненнях. Підхід базується на ідеях Харзеева Д. та ін. і може бути застосований для розв'язку проблеми термалізації.

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## ЭНТРОПИЯ ТЕРМАЛИЗАЦИИ ЧАСТИЦ И ЭФФЕКТ УНРУ

### Резюме

Предложен метод оценки энтропии, генерируемой во время разрыва струны в высокоэнергетических столкновениях. Метод основан на идеях Харзеева Д и др. и может быть применён при решении проблемы термализации.